Examiners' Report: Preliminary Examination in Mathematics Trinity Term 2017

November 8, 2017

Part I

A. STATISTICS

- **Numbers and percentages in each class.**
  See Table 1. Overall 201 candidates were classified.

Table 1: Numbers in each class (Preliminary Examination)

	Numbers					Percentages %				
	2017	(2016)	(2015)	(2014)	(2013)	2017	(2016)	(2015)	(2014)	(2013)
Distinction	62	(59)	(55)	(55)	(55)	30.85	(30.89)	(30.73)	(30.9)	(30.9)
Pass	124	(119)	(105)	(103)	(103)	61.69	(62.3)	(58.66)	(57.87)	(57.87)
Partial Pass	13	(7)	(13)	(12)	(13)	6.47	(3.66)	(7.26)	(6.74)	(7.3)
Incomplete	0	(0)	(1)	(0)	(0)	0	(0)	(0)	(0.56)	(0)
Fail	2	(6)	(6)	(7)	(7)	0.99	(3.14)	(3.35)	(3.93)	(3.93)
Total	201	(191)	(179)	(178)	(178)	100	(100)	(100)	(100)	(100)

- **Numbers of vivas and effects of vivas on classes of result.**
  As in previous years there were no vivas conducted for the Preliminary Examination in Mathematics.
- **Marking of scripts.**
  As in previous years, no scripts were multiply marked by Moderators; however all marking was conducted according to a detailed marking scheme, strictly adhered to. For details of the extensive checking process, see Part II, Section A.

B. New examining methods and procedures

No new examining methods and procedures were used for 2016/17, with the exception of a change in the wording (but not the intention) of the Examination Conventions, to clarify the requirements for a distinction.

C. Changes in examining methods and procedures currently under discussion or contemplated for the future

No changes are under discussion for 2017/18.

D. Notice of examination conventions for candidates

The Notice to Candidates, containing details of the examinations and assessment, including the Examination Conventions, was issued to all candidates at the beginning of Trinity term. All notices and the Examination Conventions in full are available at

https://www.maths.ox.ac.uk/members/students/undergraduate-courses/examinations-assessments/examination-conventions.

Part II

A. General Comments on the Examination

Acknowledgements

The Moderators express their sincere thanks to the academic administration team, and in particular to Nia Roderick and Charlotte Turner-Smith, for all their work in running the examinations system and supporting the Moderators at every turn whilst being careful always to facilitate but never to influence academic decisions made by the Moderators.

We are very grateful to Dr Andrew Thompson for administering the Computational Mathematics projects. We would also like to thank the Assessors Dr Adam Gal, Dr Stephen Haben, Dr Eoin Long, Dr Raka Mondal, Dr Sourav Mondal, Dr Stephen Muirhead, Dr Tigran Nagapetyan, and Dr Rolf Suabedissen, for their assistance with marking.

Timetable

The examinations began on Monday 19th June at 2.30pm and ended on Friday 23rd June at 11.30am.

Factors Affecting Performance

A subset of the Moderators attended a pre-board meeting to band the seriousness of circumstances for each application of factors affecting performance received from the Proctors' office. The outcome of this meeting was relayed to the Moderators at the final exam board. The moderators gave careful regard to each case, scrutinised the relevant candidates' marks and agreed actions as appropriate.

See Section E for further detail.

Setting and checking of papers and marks processing

The Moderators first set questions, a checker then checked the draft papers and, following any revisions, the Moderators met in Hilary term to consider the questions on each paper. They met a second time to consider the papers at the end of Hilary term making further changes as necessary before finalising the questions. A meeting was held in early Trinity term for a final proof read. The Camera Ready Copy (CRC) was prepared and each Moderator signed off the papers. The CRC was submitted to Examination Schools in week 4 of Trinity term.

The examination scripts were collected from Ewert House and delivered to the Mathematical Institute.

Once the scripts had been marked and the marks entered, a team of graduate checkers, under the supervision of Sandy Patel, sorted all the scripts for each paper of the examination. They carefully cross checked against the mark scheme to spot any unmarked questions or parts of questions, addition errors or wrongly recorded marks. A number of errors were corrected, with each change checked and signed by an Examiner, at least one of whom was present throughout the process. A check-sum was also carried out to ensure that marks entered into the database were correctly read and transposed from the marks sheets.

Determination of University Standardised Marks

The candidates under consideration are Mathematics and Mathematics & Statistics candidates, 201 in total. We do not distinguish between them as they all take the same papers.

Marks for each individual paper are reported in university standardised form (USM) requiring at least 70 for a Distinction, 40–69 for a Pass, and below 40 for a Fail.

As last year the Mathematics Teaching Committee issued each examination board with broad guidelines on the proportion of candidates that might be expected in each class. This was based on the average proportion in each class over the past five years, together with recent historical data for Honour Moderations.

The raw marks were recalibrated to arrive at the USMs reported to candidates, adopting the procedures outlined below. These procedures are similar to the ones used in previous years.

To ensure equal weightings across all subjects, papers were first standardised to have broadly similar proportions of candidates attaining each class. A piecewise linear mapping was adopted to produce a USM from a raw mark. The default algorithm for each paper works as follows.

1. Candidates' raw marks for a given paper are ranked in descending order. Here the population data used is the set of marks for all candidates in Mathematics or Mathematics & Statistics.

2. The default percentages p_1 of Distinctions, p_2 of nominal upper seconds (USM 60–69) and p_3 of nominal lower seconds and below in this pop-

ulation are selected, these percentages being similar to those adopted in previous years.

3. The candidate at the p_1-th percentile from the top of the ranked list is identified and assigned a USM of 70. Let the corresponding raw mark be denoted by R_1.

4. Similarly, the candidate at the $(p_1 + p_2)$-th percentile from the top of the list is assigned a USM of 60 and the corresponding raw mark is denoted by R_2. Likewise R_3 is the raw mark corresponding to the $(p_1 + p_2 + p_3)$-th percentile.

5. The line segment between $(R_1, 70)$ and $(R_2, 60)$ is extended linearly to the USMs of 72 and 57 respectively. Denote the raw marks corresponding to USMs of 72 and 57 by C_1 and C_2 respectively. Line segments are then drawn connecting $(C_1, 72)$ to $(100, 100)$.

6. Finally, the line segment through the corner at $(C_2, 57)$ is extended down towards the vertical axis as if it were to join the axis at $(0, 10)$, but is broken at the corner $(C_3, 37)$ and joined to the origin, yielding the last segment in this model. Here C_3 is obtained as above by extension from $(R_3, 40)$.

Thereby a piecewise linear map is constructed whose vertices, at $\{(0, 0), (C_3, 37), (C_2, 57), (C_1, 72), (100, 100)\}$, are located away from any class boundaries.

A first run of the outlined scaling algorithm was performed. It was confirmed that the procedure resulted in a reasonable proportion of candidates in each class. The Moderators then used their academic judgement to make adjustments where necessary as described below. The Moderators were not constrained by the default scaling map and were able, for example, to insert more vertices if necessary.

To obtain the final classification, a report from each Assessor was considered, describing the apparent relative difficulty and the general standard of solutions for each question on each paper. This information was used to guide the setting of class borderlines on each paper.

The scripts of those candidates in the lowest part of each ranked list were scrutinised carefully to determine which attained the qualitative class descriptor for a pass on each paper. The gradient of the lower section of the scaling map was adjusted to place the pass/fail borderline accordingly.

Careful consideration was then given to the scripts of candidates at the Distinction/Pass boundary.

Adjustments were made to the scaling maps where necessary to ensure that the candidates' performances matched the published qualitative class descriptors.

The Computational Mathematics assessment was considered separately. In consultation with the relevant Assessor it was agreed that no recalibration was required, so the raw marks (out of 40) were simply multiplied by 2.5 to produce a USM.

Finally, the class list for the cohort was calculated using the individual paper USMs obtained as described above and the following rules:

Distinction: both $Av_1 \geq 70$ and $Av_2 \geq 70$ and a mark of at least 40 on each paper and for the practical assessment;

Pass: not meriting a Distinction and a USM of at least 40 on each paper and for the practical assessment;

Partial Pass: awarded to candidates who obtained a standardised mark of at least 40 on three or more of Papers I-V but did not meet the criteria for a pass or distinction;

Fail: a USM of less than 40 on three or more papers.

Here Av_2 is the average over the five written papers, weighted by length, and Av_1 is the weighted average over these papers together with Computational Mathematics (counted as one third of a paper). The Moderators verified that the overall numbers in each class were in line with previous years, as shown in Table 1.

The vertices of the final linear model used in each paper are listed in Table 2, where the x-coordinate is the raw mark and the y-coordinate the USM.

Table 2: Vertices of final piecewise linear model

Paper	Positions of vertices				
I	(0,0)	(24.28,37)	(42.27,57)	(70.91,72)	(100,100)
II	(0,0)	(21,37)	(34,57)	(64,72)	(100,100)
III	(0,0)	(37.03,37)	(64.45,57)	(97.18,72)	(120,100)
IV	(0,0)	(26.69,37)	(46.45,57)	(64.18,72)	(100,100)
V	(0,0)	(20.79,37)	(36.18,57)	(55.27,72)	(80,100)
CM	(0,0)				(40,100)

Table 3 gives the rank list of average USM scores, showing the number and percentage of candidates with USM greater than or equal to each value.

Table 3: Rank list of average USM scores

		Candidates with USM $\geq x$	
USM (x)	Rank	Number	%
96	1	1	0.50
94	2	2	1.00
90	3	3	1.49
89	4	4	1.99
87	5	5	2.49
86	6	6	2.99
85	7	7	3.48
83	8	8	3.98
82	9	9	4.48
81	10	15	7.46
80	16	17	8.46
79	18	20	9.95
78	21	22	10.95
77	23	24	11.94
76	25	30	14.93
75	31	34	16.92
74	35	37	18.41
73	38	45	22.39
72	46	50	24.88
71	51	56	27.86
70	57	62	30.85
69	63	73	36.32
68	74	84	41.79
67	85	94	46.77
66	95	103	51.24
65	104	115	57.21
64	116	121	60.20
63	122	130	64.68
62	131	142	70.65
61	143	151	75.12
60	152	157	78.11
59	158	162	80.60
58	163	164	81.59
57	165	169	84.08
56	170	173	86.07
55	174	177	88.06
54	178	180	89.55
53	181	183	91.04
52	184	184	91.54

Table 3: Rank list of average USM scores (continued)

		Candidates with USM $\geq x$	
USM (x)	Rank	Number	%
50	185	187	93.03
49	188	190	94.53
48	191	193	96.02
47	194	194	96.52
43	195	195	97.01
41	196	198	98.51
39	199	199	99.00
38	200	200	99.50
36	201	201	100.00

Recommendations for Next Year's Examiners and Teaching Committee

None.

B. Equal opportunities issues and breakdown of the results by gender

Table 4 shows the performances of candidates broken down by gender.

Table 4: Breakdown of results by gender

Class	Number								
	2017			2016			2015		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	12	50	62	7	52	59	7	48	55
Pass	36	88	124	36	83	119	34	71	105
Partial Pass	4	9	13	1	6	7	4	9	13
Incomplete	0	0	0	0	0	0	0	0	0
Fail	0	2	2	3	3	6	3	3	6
Total	52	149	201	47	144	191	48	131	179

Class	Percentage								
	2017			2016			2015		
	Female	Male	Total	Female	Male	Total	Female	Male	Total
Distinction	23.08	33.56	30.85	14.89	36.11	30.89	14.58	36.64	30.73
Pass	69.23	59.06	61.69	76.6	57.64	62.3	70.83	54.20	58.66
Partial Pass	7.69	6.04	6.47	2.13	4.17	3.66	8.33	6.87	7.26
Incomplete	0	0	0	0	0	0	0	0	0
Fail	0	1.34	1	6.38	2.08	3.14	6.25	2.29	3.35
Total	100	100	100	100	100	100	100	100	100

C. Statistics on candidates' performance in each part of the Examination

The number of candidates taking each paper is shown in Table 5. The performance statistics for each individual assessment are given in the tables below: Paper I in Table 6, Paper II in Table 7, Paper III in Table 8, Paper IV in Table 9, Paper V in Table 10 and Computational Mathematics in Table 11. The number of candidates who received a failing USM of less than 40 on each paper is given in Table 5.

Note that Paper I, II and IV are marked out of 100 (being 2.5 hours in duration), Paper III is marked out of 120 (being 3 hours in duration) and Paper V is marked out of 80 (being 2 hours in duration).

Table 5: Numbers taking each paper

Paper	Number of Candidates	Avg RAW	StDev RAW	Avg USM	StDev USM	Number failing	% failing
I	201	59	14.96	65.73	10.64	4	2
II	201	51.14	15.72	65.23	10.72	4	2
III	201	81.85	18.51	65.72	11.94	7	3.4
IV	201	57.13	12.9	65.69	11.2	4	2
V	201	47.44	12.28	65.85	12.23	4	2
CM	201	36.25	5.13	90.83	12.8	2	1

Table 6: Statistics for Paper I

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	14.19	14.21	2.85	198	1
Q2	15.69	15.69	4.08	164	0
Q3	12.04	12.04	5.03	79	0
Q4	10.14	10.16	3.76	159	1
Q5	11.25	11.25	4.02	178	0
Q6	8.55	8.55	3.63	162	0
Q7	8.95	8.95	3.66	58	0

Table 7: Statistics for Paper II

Question Number	Average Mark All	Used	Std Dev	No. of Attempts Used	Unused
Q1	11.75	11.75	3.98	84	0
Q2	13.07	13.07	3.21	168	0
Q3	12.34	12.39	3.75	150	1
Q4	8.71	8.71	4.05	156	0
Q5	8.83	8.83	3.56	167	0
Q6	8.38	8.38	4.36	78	0
Q7	9.31	9.31	5.84	188	0

Table 8: Statistics for Paper III

Question Number	Average Mark All	Used	Std Dev	No. of Attempts Used	Unused
Q1	16.17	16.17	3.35	195	0
Q2	14.36	14.36	4.76	107	0
Q3	16.14	16.10	3.62	100	1
Q4	13.28	13.28	4.27	177	0
Q5	14.98	14.98	3.51	164	0
Q6	14.76	14.76	3.42	58	0
Q7	8.94	8.92	5.58	171	1
Q8	13.60	13.60	5.33	176	0
Q9	10.36	10.56	3.60	54	1

Table 9: Statistics for Paper IV

Question Number	Average Mark All	Used	Std Dev	No. of Attempts Used	Unused
Q1	10.48	10.48	3.57	142	0
Q2	10.88	10.88	4.33	94	0
Q3	13.20	13.20	2.97	165	0
Q4	8.95	8.95	3.51	98	0
Q5	14.05	14.05	3.82	196	0
Q6	9.76	9.76	5.11	98	0
Q7	10.99	10.99	3.11	201	0

Table 10: Statistics for Paper V

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	12.21	12.21	3.88	185	0
Q2	9.63	9.63	4.11	162	0
Q3	11.09	11.09	4.60	53	0
Q4	13.68	13.68	3.74	159	0
Q5	12.92	12.94	3.75	116	1
Q6	11.43	11.63	4.57	125	4

Table 11: Statistics for Computational Mathematics

Question Number	Average Mark All	Average Mark Used	Std Dev	No. of Attempts Used	No. of Attempts Unused
Q1	18.08	18.08	3.00	198	0
Q2	18.35	18.35	2.77	181	0
Q3	16.78	16.78	4.09	23	0

D. Comments on papers and on individual questions

Paper I

Question 1

(a) The definitions were straightforward. A wide variety of versions of Steinitz' Exchange Lemma were stated, but not all proofs did in fact prove all of the stated version. Often, the easy containment was missing, other times no argument was given why a scalar they divided by was non-zero. Only very few candidates applied the Lemma to (iii) and many solved this part 'by hand'. A significant number of candidates gave only proofs for one of linear independence or spanning.

(b) In part (b), a lot of answers had significant logical problems or ignored potential division by 0. A common issue was to assume $\lambda(a,b)+\mu(c,d)=0$, deduce that $\lambda(ad-bc)=0$ and then argue that if $ad-bc=0$ then the original equation must have had a non-trivial solution because the deduced equation does. A lot of candidates also used the fact that the vectors are linearly independent if and only if the determinant of the matrix with these vectors as columns was non-zero without justification. Only partial credit was given to this.

(c) Only few candidates used the results from the earlier parts. In the 'manual' solutions, often only linear independence was shown (for α, β distinct and non-zero) and any comment as to why this implied that the three vectors formed a basis was missing. A number of candidates simply computed the determinant of the 3×3 matrix with the appropriate columns and simply stated that this was non-zero if and only if the three vectors formed a basis, for which only partial credit was given.

Question 2

(a) Generally well-done, though again logical problems surfaced frequently: A common approach was to assume that x_0 and x_1 were solutions and deduce that $x_0 - x_1 \in \ker(T_A)$. It was then stated that this implied that the existence of unique solutions implied that the kernel was trivial, which goes against the logical flow of the preceding argument. Often the condition $\mathbf{b} \in \operatorname{Im}(T_A)$ was completely ignored.

(b) Giving concrete counterexamples where appropriate seemed tricky for a lot of candidates although it was the easiest approach as the zero-matrix with a non-zero $\mathbf{b}$ and the identity matrix were sufficient. A clear description of how a counterexample might be obtained was given full marks.

(c) Most candidates did this part very well, although numerous calculation errors and even errors in copying out the matrix were made. A lot of candidates considered the case $\lambda \neq 5, \mu = 2$ separately from $\lambda \neq 5, \mu \neq 2$, although their computations gave no cause for doing so.

Question 3

In this question the logical flow of the arguments was much better than in questions 1 and 2. In checking that $T^{-1}(X)$ was a subspace, a lot of candidates forgot to check for non-emptyness. Slightly concerning were the number of candidates who argued $T^{-1}(X) = X \cap \mathrm{Im}(T) \oplus \ker(T)$ although the two spaces on the RHS are subspaces of different vector spaces. Similarly a number of candidates assumed that T^{-1} was a well-defined map or tried to compute the dimension of a subset which was not a subspace.

In part (c), a number of candidates tried to used the inequalities computed earlier, and eventually used an obviously wrong inequality (e.g. $a + b \leq a$ for non-negative a, b) to make their argument work.

Question 4

Part (a) was standard bookwork and done perfectly by many candidates. However, candidates had enormous difficulties with part (b), especially with (b)(i): only two candidates managed to get (b)(i) right, only one coming up with the simple observation that if $T^2 = \mathrm{id}_V$ then $V = E_1 \oplus E_{-1}$, by writing any $v \in V$ as $v = \frac{1}{2}(v + Tv) + \frac{1}{2}(v - Tv)$. Many candidates seemed to lack a good geometric understanding, but marking of this part was generous.

Question 5

Parts (a) and (b) were easy bookwork with a high success rate. In part (c), while done correctly by a good number of candidates, often only one direction of the double implication was shown (typically the easier forward direction). In part (d) many candidates guessed the correct solution ($n = 12$), but they 'proved' it by the invalid argument that if S_n has a subgroup isomorphic to $S_4 \times S_8$ then S_n must contain a 4-cycle and a disjoint 8-cycle (for example, $S_2 \times S_2$ is isomorphic to a subgroup of A_4, namely V_4, but A_4 does not contain two disjoint 2-cycles). Some candidates thought that there must be a connection to part (c), so their guess for n was a prime power $n = 16$ or $n = 32$.

Question 6

Parts (a) and (b) were mostly bookwork or similar to exercises except for (b)(ii) where most candidates nicely proved that there are at most $p - 1$ automorphisms of C_p, but often failed to show that all 'potential' automorphisms actually occur. Part (c) was a lot more challenging. Many candidates did well on (c)(i): they either spotted the simple counting argument analogous to some bookwork examples, or (much longer) they used and proved Cauchy's Theorem. Parts (c)(ii) and (c)(iii), however, got very few correct answers. While (c)(ii) requires another slightly more involved counting argument (using (b)(iv)), the (fairly short) proof in (c)(iii) could only be carried out on the basis of a very good understanding of all the statements in all previous parts.

Question 7

Everyone knew the definition of an action. A surprisingly common mistake was then considering the regular action in the second part instead of a general one. About the morphism to $Sym(X)$: Many students claimed that the images of g and g^{-1} are inverses without proving it. There were relatively few students who wrote the whole solution in a clear and organized way.

(i) Most students easily showed that S^1 is invariant under the action, and to varying degrees succeeded in describing the stabilizer of the given vector. Several students only described the stabilizer as "rotations" around a line, without proof, for which I removed a mark. (ii) In the part about transitivity very few students gave a full satisfactory solution. There were very many hand wavy solutions again using the notion of rotations without making it precise. Most of the satisfactory solutions used the polar coordinates approach which appeared as the alternative solution in the mark scheme. (iii) In the last part about subgroups most students who attempted this found the cyclic subgroups but very few gave a construction for infinitely many non-abelian subgroups which are obviously different.

Paper II

Question 1

This question was the least popular from Section A. Part (a) was essentially bookwork and was done quite well, with most students finding (ii) a little easier than (i). A common error in both (a) and (b) was to assume that the sup must be achieved by an element from the set (i.e. sup = max). Part (b) tended to be more variable in mark distribution with many students solving (b)(i) and (ii) but less getting (iii) and (iv). While (b)(i) is true, a number of students attempted induct from (a)(ii) which doesn't work. Part (a)(i) with (b)(i) together gave a very short proof of (b)(ii). The most difficult part of the question was in proving that (b)(iii) is true, which quickly followed from the key observation that $\sup(R_m) \geq x_{m,n} \geq \inf(C_n)$ for all $m, n \in \mathbb{N}$. A number of students found a counterexample to (b)(iv) although some only gave a 2×2 array as a counterexample.

Question 2

This question was the most popular from Section A and marks were very evenly spread. Part (a) was answered correctly by almost everyone, except that some gave $a_n \neq 0$ as the desired condition rather than $L \neq 0$. Part (b)(i)-(iii) was well structured, each solvable independently from the others (for example a number solved (iii) but not (ii)). Most students spotted (i) by bringing (*) to a common denominator. Part (ii) caused more difficulty, although it followed quite easily by squaring (*) and noting what remains is c plus a squared number. Most students spotted that (i) and (ii) implied (iii). To complete (b) it was enough to note that (a_n) tends to a limit L, with $L \neq 0$ by AOL applied to (*). A small point here was that some candidates proved (a_n) converges by noting that the sequence is monotonic decreasing by (iii) and by (i) and (ii) it is bounded below by $\sqrt{c}$. The question essentially asks to prove that $\sqrt{c}$ exists, so it was better to lower bound by 0 say. For (c) many students noted that the same proof of (b)(ii) works here (or simply that $a_n^2 \geq 0 > c$) while less found counterexamples to (i) and (iii). Lastly, far fewer candidate managed to prove (ii) although a number noted that if the sequence is eventually positive then by (*) it must be decreasing (sim. if eventually negative).

Question 3

Part (a) was done quite well, but a surprising number of candidates had difficulty in proving that if (z_n) is Cauchy then (x_n) and (y_n) are also Cauchy. Many candidates proved $\sum z_n$ is Cauchy in the final part of (a) and then claimed that it is convergent without noting that this can only be used for real sequences. Part (b) was quite standard bookwork, and most students proved the absolute convergence statement, but less with the divergence statement. Part (c)(i) generally went well, provided the ratio test was used (a common error was to try to apply the alternating series test here). In (c)(ii) a small number of students assumed that the conditions of the ratio test are necessary if the series converges (which led to a radius of convergence 0 instead of 1). Most solutions split the series into two series, consisting of odd and even powers respectively and calculated the radii of convergence for each R_1 and R_2. While this works, some justification was necessary to prove that the radius of convergence $R = \min(R_1, R_2)$.

Question 4

The first part was largely well-answered, though a number of candidates unnecessarily attempted to show $f([a, b]) = [f(a), f(b)]$ assuming f was continuous. Relatively few candidates proved that the one-sided limits of f existed in part (b), but most saw how to deduce the condition for continuity at $x_0 \in (a, b)$. Many candidates reproduced proofs of the inverse function theorem in the final part, failing to deduce it from the previous parts.

Question 5

The first part was largely well-answered, though many candidates asserted that for any $\epsilon > 0$ there is a $\delta > 0$ such that for all $n \geq N$ and $|x - x_0| < \delta$

one has $|f(x)-f(x_0)|<\epsilon$. While this is in fact true given the assumption of uniform convergence, it is not an immediate consequence of the hypotheses of the question. Many candidates found a counterexample for part (*b*), though some tried examples like $f_n = \frac{1}{n}\mathbf{1}_{\{1/n\}}$, which is not in fact a counterexample since for each $x_0 \in [0,1]$ there is an N such that f_n is continuous at x_0 for all $n \geq N$. Part (*c*) was well-answered, while part (*d*) was successfully attempted by only a small number of candidates.

Question 6

For part (*a*) many candidates successfully showed the linear approximation condition implied differentiability, but fewer were able to clearly define the function $\epsilon(x)$ for the converse. In part (*b*) many candidates sensibly applied part (*a*), but fewer saw how to use the inequalities $x_n \leq c \leq y_n$ to conclude the limit exists. Despite being similar to previous exam questions and problem sheet questions, the final part was relatively poorly answered.

Question 7

Part (a) Some students quoted the theorem that continuous functions are integrable, which considering the rest of the question was very circular logic, but as there was some non-trivial argument involved I decided to give this 3/6 marks when done correctly. Otherwise many students solved this part mostly satisfactorily. A common mistake was confusing continuity and uniform continuity. So students used uniform continuity without stating that it holds in this case.

Part (b) All correct solutions involved constructing sequences of majorants and minorants as was taught this year in the course.

There was a common bad strategy of stating that the sequence of step functions can be split into two subsequences - majorants and minorants for f, which is of course not true in general.

Part (c) Very few students solved this question well.

In the part about discontinuities many students had trouble evaluating the function properly, and many had made the mistaken assumption that the rationals can be enumerated in an increasing order.

There were many solutions claiming that a jump exists but not proving it. A recurring wrong explanation involved the fact that there is an irrational between every two rationals.

In the part about integrability many students gave a good solution. Many others however claimed that the function is a step function since it is the sum of step functions.

The last part (deducing existence of a function with the given properties) was done by most students and I gave one mark for this.

Paper III

Question 1

This question was very popular and well-attempted.

Part (a) : This part was done perfectly by almost all the students. The concept was very clear to all. A few students did not consider the integration constant properly. However, everybody got the method right.

Part (b): This part was also well done by students. Some of them made errors in solving the differential equation. However, the fundamental concept was clear to everybody.

Part (c): This part was the most challenging among all. Everybody got the complementary function right. Finding the particular integral was the challenging part. Some of the students followed the right method and got the right answer. Some followed the right method but the answer was wrong. Everybody was awarded marks to the extent to which they got it right.

Question 2

The students performed reasonably well. In Q2(b) some of the students got confused with the trigometric identity substitution and the limits of the integral. Q2(c) was fairly difficult but was well attempted by most of the students.

Question 3 The students performed reasonably well. Some of the students had some difficulties in distinguishing two cases ($\lambda = 1$ and $z = 0$) for the solution of Q3(c) and therefore missed the maximum for the function. Only a couple of students could not write the Taylor series expansion of a two variable function

Question 4

Most of this question was done well by many candidates. Only a minority got to grips with (b)(iii). A common situation was to obtain the generating function of $\tilde{Y}$ involving μ, but not to notice that $\mu = 1/p$ as found in (a)(iii). Quite a few candidates wrote a plausible answer but as if the question had said "Suppose that $\tilde{Y}$ has the same distribution as X_1+X_2-1.... What is the distribution of X_1?". Finally, once obtaining that $\tilde{Y}$ and $X_1 + X_2 - 1$ have the same generating function, one should mention uniqueness of generating functions to conclude that $\tilde{Y}$ and $X_1 + X_2 - 1$ have the same distribution.

Question 5

Done well until the final part. In (b)(ii), the anticipated solution was to condition on the first two steps of the walk, but in fact the great majority of candidates conditioned instead on what happens at times $n-2$ and $n-1$, which is equally valid. The solution of the recurrence relation was done efficiently by many candidates. Only around 25% found the correct expression in (b)(iv), and even fewer took the limit correctly.

Question 6

Parts (a) and (b) were mostly done successfully (although quite a few candidates made rather heavy weather of the calculations in (a)). For the proof in (b)(i), one should make some sort of mention of linearity of expectation. The hardest part is the variance calculation in (c)(i), which requires some care to get exactly right! Most people who quoted Chebyshev's inequality correctly were successful in (c)(ii), although there was a surprising amount of nonsense written in passing from the bound on $\mathbb{P}(|S_n/n - 1| > \epsilon)$ to the bound on $\mathbb{P}(S_n/n > 1 + \epsilon)$.

Question 7

Most students correctly identified the MLE, although more had problems finding the unbiased estimator based on it. Candidates also found it more challenging to find a confidence interval with the MLE as an endpoint, but there were plenty of good answers. For the two-sided exponential distribution in the final part, a very common mistake was to assume that the minimiser for the sum of absolute values is the same as that for the sum of squares.

Question 8

The majority correctly derived the estimators (although there were some mistakes in the case of $\hat{\beta}_1$, mostly since students forgot to square the denominator when estimating the variance). Many answers to the second part were also broadly good, although there were many small errors or omissions in formulating the CLT and in justifying its application to the distribution of $\hat{\beta}_0$. In the final part, many candidates missed the point that the terms in the sum giving $\hat{\beta}_1$ are no longer identically distributed, so that the CLT does not apply directly.

Question 9

This question was not popular (perhaps many candidates were not prepared for the somewhat free-form style). Most people who answered it had a plausible idea of the steps of the algorithm, although some accounts were much more convincing than others. Many did not accurately write down the objective function. Almost nobody gave a convincing argument for the termination of the algorithm. (Quite a few observed that the objective function decreases as the algorithm proceeds, and is bounded below, and so it must converge, but convergence is not the same as termination.) As a consequence, part (c) was out of reach of most of the candidates, and very few picked up any marks there; only one candidate found n^{K-1} as a bound on the number of possible partitions into K clusters.

Paper IV

Question 1

The first two parts of Question 1 were mostly ok, though disappointingly many candidates made fundamental mistakes such as writing that 3 vectors are linearly dependent if and only if one is a multiple of another. The last part proved to be tricky.

Question 2

Question 2 was least popular. Part (a) (bookwork) was mostly fine, though with some unclear logic. Part (b) is very similar to an example in the notes, but despite the hint many candidates completely missed the point, reducing to an equation in just x and y. This is equivalent to finding the projection of C onto the x-y plane, which has a different shape to C itself. Limited partial credit was awarded to those candidates who did this correctly.

Question 3

Question 3 was by some way the most popular. (a), which is mostly calculation, was completed successfully by the vast majority of candidates. The same applies to much of (b), though many got the last part wrong. (c) turned out to be rather difficult, with very few correct answers. A correct guess earned partial credit, but there weren't that many of those either.

Question 4

This question was done rather poorly. A common confusion in part (a) was to think that the equilibrium positions occurred at the local maxima of $f(x)$, while this is only the case for the specific value $\ell = \ell_1$, and not for $\ell > \ell_1$. Determining the stability of the steady states therefore caused some difficulty, with many candidates considering $f''(x)$ (presumably confusing this for a potential). The first part of (b) was done well by many candidates, but the second part to show whether the particle would reach the origin was very poorly done; very few candidates thought to look at the zeros of the right hand side of the given expression. A common mistake for the last part was to describe the (nonlinear) oscillations as 'simple harmonic motion'.

Question 5

This was the most popular dynamics question and was attempted by almost every candidate. Most candidates answered the bookwork in part (a) well, and part (b) quite well. Some of the justifications for the value of h lacked sufficient explanation. Common errors were to assume the perpendicular distance $r = d$ as an initial condition, and to take the radial velocity $\dot{r}$ as positive in the initial condition for $du/d\theta$. For the final part of question, many candidates did not realise that the spacecraft would have to go three-quarters of the way around the planet in order to be deflected by an angle $\pi/2$.

Question 6

This question was done well by a few candidates, but caused difficulty for many. A common mistake in part (a) was to think that energy is conserved, or to assume that the normal reaction is perpendicular to the velocity in the inertial frame (rather than perpendicular to the tangent of the wire). Many candidates therefore had to effect surreptitious sign changes to reach the given formula. Of the candidates who chose to work in a rotating frame, a common mistake was to 'double count' and include $\dot{\theta}$ terms in the rotating frame as well as in the centrifugal term. Part (b) was done surprisingly badly. The fact that the equation could not readily be expressed in the form $\ddot{r} = f(r)$ seemed to be the chief difficulty (though many simply wrote it as such anyway and did a variety of creative things with the $\dot{r}^2$ term). Many candidates found the correct criteria but did not sufficiently explain their reasoning. In part (c) it was again common to assume that energy is conserved for the first part, but the last part deducing the speed was done well.

Question 7

The first part of this compulsory question on Euclid's Algorithm was reasonably well done by the large majority of candidates, though some were extremely brief in their description for the first part; it was clear that some were hurriedly attempting this question under considerable time pressure at the end of the exam. The second part, on Newton's method, was completed very well by a few, but many did not notice that the indicated root of the polynomial was evidently also a root of its derivative and so failed to simplify the expression for g. This made the task of bounding the derivative more challenging.

Paper V

Question 1

The most popular question. The first part did not generally cause difficulty, though further into the question standard, but non-trivial, integrals caused weaker students difficulties. That Green's theorem in the plane provides a rapid means of answering the final part was often missed, but students who carefully accounted for the symmetries of the problem were nonetheless able to obtain full marks on this part.

Question 2

This was also popular. Most, but not all, candidates found it difficult to accommodate the impact of a spatially varying conductivity for the uniqueness proof of the steady state heat equation.

Question 3

This was unpopular and attempted only by a minority. Many candidates struggled to do standard, but messy, integrations with a number of physical constants under exam pressure. However, candidates should have been alert to the fact the question was phrased with physically consistent dimensions and thus dimensions could have been used to check solutions *very* quickly and thus to spot, and correct, a wide variety of errors.

Question 4

Generally Q4 (a)(i) done poorly most using an intuitive explanation.
(a)(ii) and (b) were ok.
(c) was generally ok, not everyone noticed that $a_n = 0$ due to "oddness".
(d) completed with confusion
- especially if (c) is incorrect
- convergence reasoning not clear

Question 5

This question on the Wave equation and D'Alembert's solution was attempted by nearly 2/3 of candidates. The first bookwork part was generally answered well, though a few attempted to merely demonstrate that the given form was a solution. The second part on interpreting and applying D'Alembert's solution was not so well done; there were a wide range of attempts, some scoring full marks and most gaining some credit.

Question 6

The first bookwork part of the question was done well by the vast majority. In part (b), many failed to observe the inhomogeneity of the boundary condition $T(0,t) = T_0 > 0$ and thus did not have homogeneous conditions when seeking a separation of variables solution. Several tried to compute the Fourier sine series for $(1 + x/L)T_0$ even though T_0 was part of the solution as indicated. A few produced perfect solutions to this question, securing full marks.

Computational Mathematics

The students chose two projects out of three (two Matlab-based, Projects A and B; one Sage-based, Project C), and each was marked out of 20, giving a total of 40. The majority of students scored 30 or above. Assessment was based entirely on published reports. Project A (Newton's Method and GPS) and Project B (Prony's method) were the most popular. Project C (Interpolation and the Cayley–Bacharach Theorem) was least popular. Likely reasons for the limited take-up of Project C are that students had limited exposure to Sage in advance, and also it seems that this year Project C was considered to be more challenging than in previous years. Both Projects A and B focused largely on numerical calculation, which is where Matlab is most likely useful for mathematicians. The marks for each of the projects were comparable, with the marks for projects A and B on average slightly higher. In Projects A and B, two marks were awarded for a coherently written report and well-written code, while in Project C a single mark was awarded for a well-presented report; not all students earned these marks.

Statistics by project:

Project	Take-up	Average	SD
A	198	18.12	2.99
B	181	18.37	2.76
C	23	16.78	4.00

Student scores out of 40:
Average: 35.78
SD: 6.70
Maximum: 40
Minimum: 11

E. Comments on performance of identifiable individuals

Removed from public version.

F. Names of members of the Board of Examiners

- **Examiners: Prof. Oliver Riordan (Chair), Prof. Eamonn Gaffney, Prof. Ian Hewitt, Prof. Jochen Koenigsmann, Prof. James Martin, Prof. Kevin McGerty, Prof. Andy Wathen.**
- **Assessors: Dr Adam Gal, Dr Stephen Haben, Dr Eoin Long, Dr Raka Mondal, Dr Sourav Mondal, Dr Stephen Muirhead, Dr Tigran Nagapetyan, and Dr Rolf Suabedissen.**